

OPCIÓN A

$$1. \quad A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 1 & 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 0 & -1 \\ 2 & 3 & 2 \\ 0 & 0 & 2 \end{pmatrix} \quad C = \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}$$

$$a. \quad C \begin{pmatrix} x \\ -y \end{pmatrix} = \begin{pmatrix} 1 & x \\ y & -1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ -y \end{pmatrix} = \begin{pmatrix} 1 & x \\ y & -1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$-2x - 3y = -1 + x \Rightarrow -3x - 3y = -1$$

$$x + y = -y - 1 \Rightarrow x + 2y = -1$$

$$-3x - 3y = -1$$

$$\underline{3x + 6y = -3}$$

$$3y = -4 \Rightarrow y = -\frac{4}{3}$$

$$x = -1 - 2y = -1 + \frac{8}{3} = \frac{5}{3}$$

$$\text{Os valores son } x = \frac{5}{3}, y = -\frac{4}{3}$$

b.

$$|A| = \begin{vmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 18 + 1 - 3 \neq 0 \Rightarrow \text{Rg}(A) = 3$$

$$|B| = \begin{vmatrix} 2 & 0 & -1 \\ 2 & 3 & 2 \\ 0 & 0 & 2 \end{vmatrix} = 12 \neq 0 \Rightarrow \text{Rg}(B) = 3$$

$$c. \quad X + A^t = 2I + B \Rightarrow X = 2I + B - A^t$$

$$d. \quad X = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 0 & -1 \\ 2 & 3 & 2 \\ 0 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 0 & 1 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -2 \\ 1 & 2 & 1 \\ 0 & -1 & 2 \end{pmatrix}$$

$$2. \quad V(t) = \begin{cases} 12t - t^2 & 0 \leq t \leq 7 \\ t^2 - 18t + 112 & 7 < t \leq 12 \end{cases}$$

a. $0 \leq t \leq 7 \quad V(t) = 12t - t^2$

$$V'(t) = 12 - 2t = 0 \Leftrightarrow t = 6$$

$$V''(t) = -2 < 0 \Rightarrow \text{En } t = 6 \text{ meses hai un máximo.}$$

$$V(6) = 12 \cdot 6 - 6^2 = 36$$

Polo tanto, as vendas máximas foron de 36.000 unidades e alcanzáronse no sexto mes.

Para $7 < t \leq 12$, $V(t) = t^2 - 18t + 112$

$$V'(t) = 2t - 18 = 0 \Leftrightarrow t = 9$$

$$V(9) = 31 \quad V''(t) = 2 > 0 \Rightarrow \text{En } t = 9 \text{ hai un mínimo.}$$

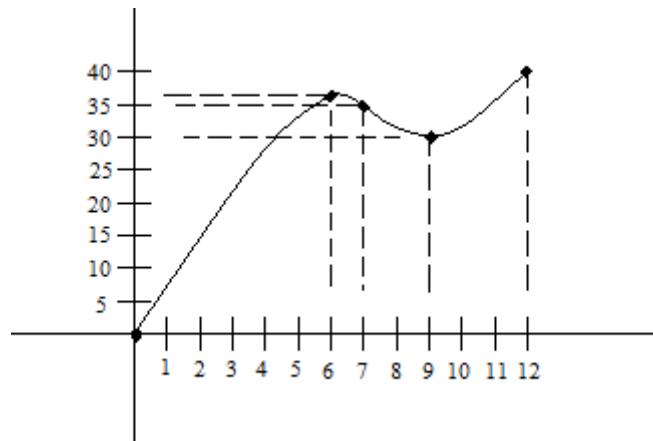
t	$t^2 - 18t + 112$
7	35
12	40

Calculamos os puntos de corte co eixe OX :

$$0 = t^2 - 18t + 112 \Rightarrow t = \frac{18 \pm \sqrt{18^2 - 4 \cdot 112}}{2} = \frac{18 \pm \sqrt{-124}}{2} \quad \cancel{\neq}$$

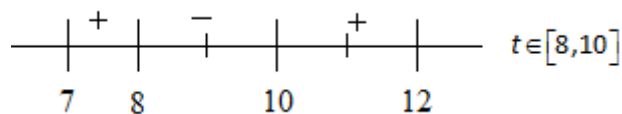
Para $V(t) = 12t - t^2$ calculamos os puntos:

x	y
0	0
7	35



b. $V(t) = t^2 - 18t + 112 \quad 7 < t \leq 12$
 $V(t) \leq 32 \Rightarrow t^2 - 18t + 112 \leq 32 \Rightarrow t^2 - 18t + 80 \leq 0$

$$t = \frac{18 \pm \sqrt{18^2 - 4 \cdot 80}}{2} = \frac{18 \pm 2}{2} \begin{cases} t=10 \\ t=8 \end{cases}$$



Pelo tanto, as vendas são menor ou igual a 32.000 unidades do mês 8 ao mês 10.

3.

$$p(H) = 0.30 \quad P(M/E) = 0.40$$

$$p(E) = 0.25$$

a. $P(M \cap E)$

$$P(M/E) = \frac{P(M \cap E)}{P(E)} = 0,40 \rightarrow P(M \cap E) = 0,40 \cdot P(E)$$

$$P(M \cap E) = 0,40 \cdot 0,25 = 0,10$$

b. $P(\bar{E}/H) = \frac{P(\bar{E} \cap H)}{P(H)} = \frac{0,15}{0,30} = 0,5$

$$P(\bar{E} \cap H) = P(H) - P(E \cap H) = 0,30 - 0,15 = 0,15$$

$$P(E) = P(H \cap E) + P(M \cap E) = 0,25$$

$$P(H \cap E) + 0,10 = 0,25 \Rightarrow P(H \cap E) = 0,15$$

4. $X =$ "nº de años de vida útil"

$$X \sim N(\mu, 1'6)$$

a. $n = 100$, $\bar{X} = 4'4$ 95% confianza

$$I_{\mu} \left(\bar{X} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{X} + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right)$$

$$I_{\mu} = \left(4'6 - 1'96 \frac{1'6}{\sqrt{100}}, 4'6 + 1'96 \frac{1'6}{\sqrt{100}} \right)$$

$$I_{\mu} = (4'2864, 4'9136)$$

b. $X \sim N(4'16, 1'6)$ $n = 64$

$$\bar{X} \sim N\left(4'16, \frac{1'6}{\sqrt{64}}\right) = N(4'6, 0'2)$$

$$P(4'25 < \bar{X} < 4'95) = P\left(\frac{4'25 - 4'6}{0'2} < Z < \frac{4'95 - 4'6}{0'2}\right) =$$

$$= P(-1'75 < Z < 1'75) = P(Z < 1'75) - (1 - P(Z < 1'75)) =$$

$$= 0'9599 - (1 - 0'9599) = 0'9198$$

OPCIÓN B

1. $f(x,y) = 2x - 3y$

a.
$$\begin{cases} x + 2y \leq 40 \\ x + y \geq 5 \\ 3x + y \leq 45 \\ x \geq 0 \end{cases}$$

$$\begin{aligned} x + 2y &= 40 \\ 2y &= 40 - x \\ y &= 20 - \frac{x}{2} \end{aligned}$$

$$\begin{aligned} x + y &= 5 \\ y &= 5 - x \end{aligned}$$

$$\begin{aligned} 3x + y &= 45 \\ y &= 45 - 3x \end{aligned}$$

x	y
5	0
10	-5

x	y
5	30
10	15

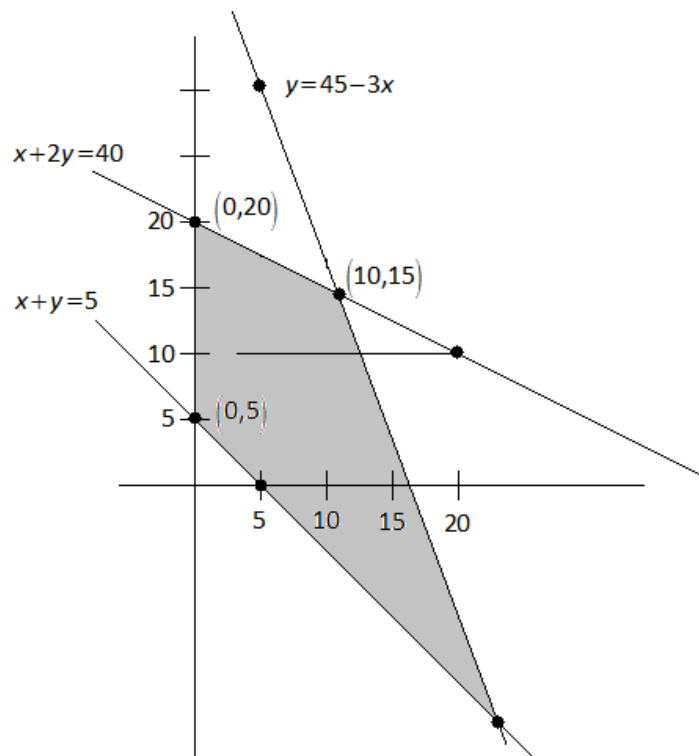
x	y
10	15
20	10

$$\begin{cases} y = 45 - 3x \\ x + 2y = 40 \end{cases}$$

$$\begin{aligned} 3x + y &= 45 \\ -3x + 6y &= 120 \\ \hline -5y &= -75 \\ y &= 15 \end{aligned}$$

$$3x + 15 = 45$$

$$3x = 30 \Rightarrow x = 10$$



$$\begin{cases} y = 45 - 3x \\ y = 5 - x \end{cases} \quad \boxed{y = -15} \quad \begin{array}{l} 45 - 3x = 5 - x \\ -2x = -40 \\ \boxed{x = 20} \end{array}$$

Os vértices son os puntos $(20, -15)$, $(10, 15)$, $(0, 20)$, $(0, 5)$

b.

$$f(20, -15) = 2 \cdot 20 - 3(-15) = 85$$

$$f(10, 15) = 2 \cdot 10 - 3 \cdot 15 = -25 \quad f(0, 5) = 2 \cdot 0 - 3 \cdot 5 = -15$$

$$f(0, 20) = 2 \cdot 0 - 3 \cdot 20 = -60$$

A función alcanza o valor máximo no punto $(20, -15)$ e o valor mínimo no punto $(0, 20)$

2. $B(x) = ax^3 - 3x^2 + bx$, $0 \leq x \leq 7$

a. $B(2) = 8$

$$B'(2) = 0$$

$$B(2) = 8a - 12 + 2b = 8 \Rightarrow 8a + 2b = 20$$

$$B'(x) = 3ax^2 - 6x + b$$

$$B'(2) = 12a - 12 + b = 0 \Rightarrow 12a + b = 12$$

$$8a + 2b = 20$$

$$b = 12 - 12a$$

$$-24a + 2b = 24$$

$$b = 12 - 12 \cdot \frac{1}{4} = 12 - 3 = 9$$

$$-16a = -4 \quad \boxed{a = \frac{1}{4}}$$

$$\boxed{b = 9}$$

b. $B(x) = 0$

$$\frac{1}{4}x^3 - 3x^2 + 9x = 0$$

$$x \left(\frac{1}{4}x^2 - 3x + 9 \right) = \begin{cases} x = 0 \\ \frac{1}{4}x^2 - 3x + 9 = 0 \end{cases}$$

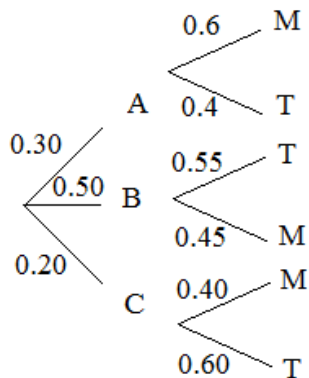
$$x = \frac{3 \pm \sqrt{9 - 4 \cdot \frac{1}{4} \cdot 9}}{2 \cdot \frac{1}{4}} = \frac{3 \pm 0}{\frac{1}{2}} = 6$$

A empresa non tivo beneficios no sexto ano.

$$\int_0^6 B(x) dx = \int_0^6 \left(\frac{1}{4} x^3 - 3x^2 + 9x \right) dx = \left[\frac{1}{4} \cdot \frac{x^4}{4} - x^3 + \frac{9x^2}{2} \right]_0^6$$

$$= \frac{1}{16} \cdot 6^4 - 6^3 + \frac{9 \cdot 6^2}{2} = 1242$$

3.



a. $P(M) = 0,30 \cdot 0,60 + 0,50 \cdot 0,45 + 0,20 \cdot 0,40 = 0,485$

O 48,5% das vendas realízase pola mañá

b. $P(C/T) = \frac{P(C \cap T)}{P(T)} = \frac{0,20 \cdot 0,6}{1 - 0,485} = 0,233$

4. 97,56% confianza

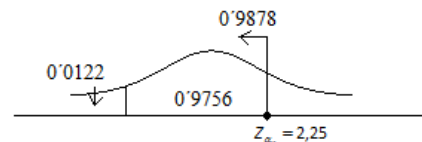
$$I_p = (0,575, 0,625)$$

a. $\hat{p} = \frac{0,575 + 0,625}{2} = 0,6$

$$E = Z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}} = 0,025$$

$$E = 2,25 \sqrt{\frac{\frac{1}{2} \cdot \frac{1}{2}}{n}} = 0,025 \Rightarrow \sqrt{\frac{1/4}{n}} = 0,0111 \Rightarrow$$

$$\frac{1/4}{n} = 0,00012345 \Rightarrow \boxed{n = 2025}$$



- b. $\hat{p} = 0,6 \Rightarrow \frac{m}{2025} = 0,6 \Rightarrow m = 1215$ pessoas contestaron que a principal causa dos accidentes é o alcohol e as drogas.